

## 2.2 Analyze Conditional Statements

Conditional statements are logical statements with two parts. When written in the *if, then* form, the "if" part contains the hypothesis and the "then" part contains the conclusion.

If p, then q.      p-hypothesis      q-conclusion

$p \rightarrow q$       "if p, then q" or "p implies q"

Examples:

If Cinderella completes her chores, then she can go to the ball.

If an angle is a right angle, then it measures  $90^\circ$ .

If a polygon has exactly 6 sides, then it is a hexagon.

Examples:

All squares are rectangles.

If it is a square,  
then it is a rectangle.

All cats are animals.

If it is a cat, then  
it is an animal.

## Related conditionals

Negation-opposite meaning as well as opposite truth value  
(Symbol  $\sim$ )

- Conditional  $p \rightarrow q$  *If p, then q.*
  - Converse  $q \rightarrow p$  *If q, then p.*
  - Inverse  $\sim p \rightarrow \sim q$  *If not p, then not q.*
  - Contrapositive  $\sim q \rightarrow \sim p$  *If not q, then not p.*
- If a conditional is true, then the contrapositive must also be true. They are said to be logically equivalent. The same is true for the converse and the inverse.

Example: Put the following statement into the if, then form. Write each related conditional. Determine whether it is True or False. If false, provide a counterexample.

Example:

All birds are owls.

Conditional:

*If it is a bird, then it is an owl.* F Robin

Converse:

*If it is an owl, then it is a bird.* T

Inverse:

*If it is not a bird, then it is not an owl.* T

Contrapositive:

*If it is not an owl, then it is not a bird.* F pig

Example: Write each related conditional. Determine whether it is True or False. If false, provide a counterexample.

If two angles form a linear pair, then they are adjacent angles. T

Converse:

*If 2  $\angle$ s are adj., then they form a L.P.* F

Inverse:

*If 2  $\angle$ s do not form a L.P., then they are not adj.  $\angle$ s.* F

Contrapositive:

*If 2  $\angle$ s are not adj., then they are not a L.P.* T

Example:

All whales are mammals.

Conditional:

Converse:

Inverse:

Contrapositive:

Biconditional Statement--conjunction of a conditional and its converse

iff

"if and only if"

Example:

If a quadrilateral has 4 right angles, ~~then~~ it is a rectangle.  
If a quadrilateral is a rectangle, then it has 4 right angles

A quadrilateral has 4 right angles iff it is a rectangle.

## 2.3 Apply Deductive Reasoning

Deductive Reasoning--use facts, rules, definitions, properties, or the laws of logic to reach logical conclusions

Example:

If 2 angles are vertical, then they are congruent.

a. Given:  $\angle 1$  and  $\angle 2$  are vertical

Conclusion:  $\angle 1 \cong \angle 2$

b. Given:  $\angle 1 \cong \angle 2$

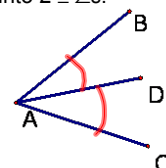
Conclusion: no conclusion

Example:

If a ray bisects an angle, then it divides it into 2  $\cong$   $\angle$ s.

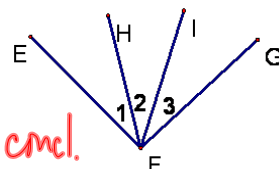
a. Given:  $\overrightarrow{AD}$  bisects  $\angle BAC$

Conclusion:  $\angle BAD \cong \angle DAC$



b. Given:  $\angle 1 \cong \angle 3$

Conclusion: no valid concl.



Example:

If a figure is a rectangle, then opposite sides are congruent.

a. Given: ABCD is a rectangle

Conclusion:

$$\overline{AB} \cong \overline{CD} ; \overline{AD} \cong \overline{BC}$$



b. Given: MNOP is a valid trapezoid

Conclusion:

no concl.

c. Given: Figure RSTU;  $RS = TU$ ,  $ST = RU$

Conclusion:

no concl.



Examples of Law of Detachment

Law of Detachment--If  $p \rightarrow q$  is true, and  $p$  is true then  $q$  is true.

If given matches hypothesis then you can make a concl.

Different type of reasoning.

Example:

Given:  $\overline{WX} \cong \overline{UV}$ ;  $\overline{UV} \cong \overline{RT}$

Conclusion:

$$\overline{WX} \cong \overline{RT}$$

Example:

(1) If Casey gets to bat, then he will get a hit.

(2) If Casey gets a hit, then we will win the game.

(3) If Casey gets to bat, then we will win the game.

These are examples of the Law of Syllogism

If  $p \rightarrow q$ , and  $q \rightarrow r$  are true, then  $p \rightarrow r$  is true.

HW

p82-83

#s 1, 7, 11-15, 19, 20

p90

#s 4, 5, 7, 9, 12, 13, 15